MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

45. Proposed by EDWARD R. ROBBINS, Master in Mathematics and Physics, Lawrenceville School, Lawrenceville, New Jersey.

Required several numbers each of which, when divided by 10 leaves a remainder 9; by 9 leaves 8; by 8 leaves 7; by 7 leaves 6; and so on. Also find the least such number which, when divided by 28 leaves 27; by 27 leaves 26; by 26 leaves 25; by 25 leaves 24, *et cetera ad unum.*

I. Solution by M. W. HASKELL, A. M., Ph. D., Department of Mathematics, University of California, Berkley, Cal.; NELSON L. RORAY, Professor of Mathematics in South Jersey Institute, Bridgeton, N. J.; A. H. BELL, Hillsboro, Ill., and H. C. WILKES, Skull Run, W. Va.

The problem can be also stated as follows: Required several numbers each of which, divided by 10, 9, 8, and so on, leaves a remainder (-1).

If then L be the least common multiple of 10, 9, 8 and so on, all numbers of the form $kL-1$, where k is any integer, will have the required character.

Now the least common multiple of 10, 9, 8, ...2, 1 is 2520. The required numbers are then $(2520\,k-1)$ *e. g.*, 2519, 5039, 7559, 10079, etc.

The second problem is solved in exactly the same way. The least common multiple of 28, 27, 26, ...2, 1 is 80313433200. So the required number is one less, or 80313433199.

II. Solution by the PROPOSER.

One less than the product of any number of factors will be divisible by any of the factors, or products of any or all of them, with a remainder one less than the divisor. Because $ab^2c^2d^3-1$ divided by acd gives b^2cd^2-1 for quotient and $acd-1$ for remainder. Thus, the different factors occurring in the natural numbers 1, 2, 3, etc., to 10, are (1.2.2.2.3.3.5.7), one less than the product of which is 2519, which leaves remainders less by unity than the divisors when divided by numbers 1, 2, 3, ...10. All multiples (diminished by one) of the continued product of these factors will satisfy the same demands of the problem, to-wit: 7559, 10079, 12599, etc., etc., *ad libitum.*

The factors occuring in numbers 1......28 are (1.2.2.2.2.3.3.3.5 5.7.11. 13.17.19.23) and one less than their continued product gives 80313433199, the number required.

NOTE. Of course the same numbers will accomodate 5 and 6; 9 and 10; 11 and 12; 13, 14, and 15; 17 and 18; 19, 20, 21, and 22; 23 and 24; 25 and 26; 27 and 28; and so on.

III. Solution by JOSIAH H. DRUMMOND, Portland, Maine.

I. $10a+9$ answers the first condition; multiply this by 9 and add 8, and

we have $90a+89$; proceeding in the same manner we finally have $3,628,800a+3,628,799$, in which a may be zero or any number.

II. Or, in the process as above, we may leave out factors of numbers already used and we reach the result $2520a+2519$, in which a may be zero or any number ; if a=zero, we have 2519, the smallest number that will answer the conditions of the first question.

III. It is manifest that if we take 1 from a number divisible by all the given divisors, the remainder when divided by those divisors will always leave a remainder one less than the divisor. Hence the least common mnltiple of the given divisors, less 1, is the number required. Hence, omitting the common factors in the second part of the question, we have $28.27.26.25.23.22.19.17\ldots.1=80,313,433,199$, the number required.

IV. Solution by W. H. CARTER, Professor of Mathematics, Centenary College of Louisiana, Jackson, La.

Let $10x_{10}+9$=the number, also, $9x_9+8$; $8x_8+7$; $7x_7+6$; $6x_6+5$; and so on to $2x_2+1$=the number.

$\therefore\ 9x_9+8=10x_{10}+9.\quad \therefore\ x_9=x_{10}+\frac{1}{9}(x_{10}+1).$

But x_9 and x_{10} are both integral.

$\therefore\ (x_{10}+1)/9=m$ an integer. $\quad\therefore\ x_{10}=9m-1=(90m/10)-1.$

The value of x_9 from the above equation is $(90m/9)-1$.

Similarly for the other values, the expression $x_n=(90m/n)-1$, giving one of the values for each value of n from 10 to 2. But since all these values are to be integral, $90m$ must be a multiple of each of the natural numbers from 2 to 10 inclusive. This requires m to be $4\times7=28$, or some multiple of 28. If $m=28$, $x_{10}=251$.

$\therefore\ 10x_{10}+9=2519$=one of the numbers.

Taking m=the multiples of 28, we get other numbers, 5039, 7559, 10,079. Still other numbers can be obtained by taking the higher multiples of 28 for m.

A similar solution gives for second statement, the number 80,313,433,199.

V. Solution by O. W. ANTHONY, M. Sc., Columbian University, 1702 S Street, Washington, D. C.

The problem in question may be generalized thus : Find a number such that if it be divided by a particular number or any number less than this number the remainder will be one less than the divisor.

Let x be the required number. It is evident, if k and $k+l$ be any two numbers less than the first divisor in question, the following conditions must be satisfied :

$$x/k=u_1+(k-1)/k\ldots\ldots(1).\qquad k/(k+l)=u_2+(k+l-1)/(k+l)\ldots\ldots(2),$$

$$\text{or } x=ku_1+k-1\ldots\ldots\ldots\ldots(3),\quad\text{and } x=(k+l)u_2+k+l-1\ \ldots\ldots\ldots\ldots(4).$$

Take the value of x given in (3) and substitute it for u_2 in (4). Then

$x=(k+l)[ku_1+k-1]+k+l-1\ldots\ldots(5)$, which may be reduced to the following form: $x=k[(k+l)u_1+k+l-1]+k-1\ldots\ldots(6)$.

Thus (5) and (6), which are identical, contain both the forms (3) and (4). Thus if we substitute in the manner indicated the result will contain two original forms. Some special forms required by the problem in question are:

$x=2u_1+1\ldots(1)$; $x=3u_2+2\ldots(2)$; $x=4u_3+3\ldots(3)$; $x=5u_4+4\ldots(4)$;

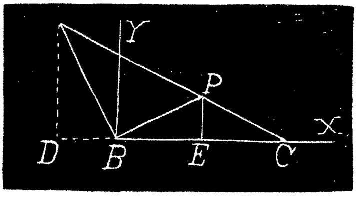
etc., etc. Substitute (1) in (2) in the manner indicated above and we have $x=6u_1+5$. This includes (1) and (2). Substitute this in (3); the result is $x=24u_1+23$. This includes (1), (2), and (3) by the previous demonstration. Continuing this we have as a result $x=\underline{|k}u_1+\underline{|k}-1=\underline{|k}(u_1+1)-1\ldots\ldots(A)$. This contains forms (1), (2), (3), (4), etc., and is the general form of number required. The examples cited are special applications of this general form. Thus $x=\underline{|8}(u_1+1)-1$ contains all the numbers required in the first part of the problem, and, letting $u_1=0$, and $k=25$, we have $x=\underline{|25}-1$, the number required in the last part of the problem.

46. Proposed by A. H. HOLMES, Box 963, Brunswick, Maine.

The base BC of the triangle ABC is $2c$, the sum of the two sides, AB and BC, is $2a$. BP is always perpendicular to AB and cuts AC in P. What is the locus of the point P?

I. Solution by GEORGE LILLEY, Ph. D., LL. D., 394 Hall Street, Portland Ore.

Take BC for the axis of x; let P be (x, y); draw AD at right angles to BC, produced; and PE at right angles to BC.



Area ABP+area PBC=area ABC, or

$(a-c)\sqrt{x^2+y^2}+cy=c\times AD\ldots\ldots\ldots\ldots(1)$.

Triangles ABD and BPE are similar.

Hence, $AD=[2y(a-c)]/\sqrt{x^2+y^2}\ldots\ldots(2)$.

From (1) and (2), $(a-c)(x^2+y^2)+cy\sqrt{x^2+y^2}=2cy(a-c)$.

$\therefore\ c^2y^2(x^2+y^2)=(a-c)^2(2cy-x^2-y^2)$, for the required locus.

If $\angle ABC$ be an acute angle, y must be taken negatively. Then, area ABC +area BPC=area ABP, or

$$c\times AD+c(-y)=(a-c(\sqrt{x^2+y^2}\ldots\ldots\ldots\ldots\ldots\ldots(3),$$

$$\text{and } AD=[2x(a-c)]/\sqrt{x^2+y^2}\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

From (3) and (4), $c^2y^2(x^2+y^2)=(a-c)^2(2cx-x^2-y^2)^2$.